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Foreign Reserves as Hedging Instruments in Emerging Countries

Summary: Emerging markets in the last decade increased the stock of foreign reserves and simultaneously managed to raise GDP growth while leaving short term foreign debt and investment in net fixed capital nearly unchanged. This work builds a model able to derive these facts as the result of greater openness to global goods and financial markets. Emerging countries generate the observed high ratios of reserves to short term foreign debt to hedge against volatility of foreign capital inflow with the purpose of stabilising not the short term but the long term finance available to domestic firms. Numerical simulations of the model derive the rising level of reserves to short term foreign debt ratio and about half of the observed rise in GDP growth as a result of a falling cost of long term finance and the increasing competitiveness of domestic industry.

Key words: Foreign reserves, Short term foreign debt, Long term finance, Growth, Investment.

JEL: F32, F36, F43, G32.

This study proposes a new approach to explaining why emerging countries, especially those that exhibit high GDP growth rates, are responsible for massive accumulation of foreign reserves.

In the aftermath of the East Asian crisis, economists and international institutions argued that the stock of international reserves, in the absence of an international lender of last resort, is warranted as insurance against foreign short term debt withdrawal (Guillermo Calvo 1998; Steven Radelet and Jeffrey D. Sachs 1998; Martin Feldstein 1999). Both the Federal Reserve and the IMF, since then, have recommended that countries follow the so called Greenspan-Guidotti rule of thumb, according to which an adequate level of reserves should be equal to the stock of the short term external debt (Alan Greenspan 1999; Stanley Fisher 2001). Several recent studies, however, have reported that the average demand for international reserves from emerging economies, which was comparable to developed countries in the 80s, has climbed since the 90s to reach levels never seen before, well above the coverage ratio recommended by the Greenspan-Guidotti rule.

With the exception of Michael B. Devereux and Alan Sutherland (2009), who argue that holding fixed income nominal bonds and issuing claims on capital (FDI) achieves a considerable degree of international risk-sharing, the prevalent opinion in the literature is that the observed stocks of reserves are not optimally determined. Richardo J. Caballero and Stavros Panageas (2004, 2005) suggest that holding state contingent assets in a Central Bank's portfolio would be a more efficient self insuring policy. Bruce Greenwald and Joseph E. Stiglitz (2010) claim that the increasing reserve hoarding is responsible for global imbalances (as savings from emerging economies could be better employed to finance domestic and global growth instead of the US current account deficit). Oliver Jeanne (2007) points out that emerging countries less exposed to the risks of the capital account crisis are those that accumulate more reserves. The evidence summarised in Section 1 of this study, however, draws attention to some stylised facts suggesting that the extent of reserves, rather than being suboptimal, is still partially unexplained. Emerging countries in the last decade seem to have reduced their exposure to the risk of short term foreign capital outflow: BRIC countries, which are the biggest and fastest growing economies, have managed to almost double their average growth rates with little growth in new capital assets and short term foreign debt; similarly, smaller emerging economies have even managed to increase growth despite a lower level of investment in new capital assets and a lower short term foreign debt. Nevertheless, emerging countries - especially those growing more - have kept raising foreign reserves massively.

This work builds a model that is able to derive both the high GDP growth and the growing stock of reserves as the result of the same process of the globalisation of the emerging economies.

The rise in the demand for international reserves has stimulated a resurgence of the literature on reserves adequacy. Earlier models on reserves adequacy date back to the1960s and 1970s and are surveyed in Robert Flood and Nancy P. Marion (2002). A mercantilist view, advanced by Michael P. Dooley, David Folkerts-Landau, and Peter Garber (2003), suggests that the accumulation of reserves in an emerging economy is the consequence of promoting export-led growth by maintaining a large and persistent current account surplus with an undervalued currency. However, as Joshua Aizenman and Jaewoo Lee (2007) argue, this approach could hardly explain why reserve accumulation grew in the last decade, while the export led growth in East Asia has been a well established strategy for the last 50 years. Moreover, the increasing reserve accumulation is carried out even in countries that have not adopted specific export-led policies and that have been using accumulated reserves to carry out countercyclical policies (André M. Cunha et al. 2011). An alternative approach, labelled the precautionary (or self-insurance) view, relates the increasing reserves to the recent financial liberalisation. This work is a further contribution to this line of research. Recent empirical studies on the effects of financial crises on output losses are Michael Hutchison and Ilan Noy (2006), Michael D. Bordo, Christopher M. Meissner, and David Stuckler (2010). The studies of Dietrich Domanski, Ingo Fender, and Patrick McGuire (2011) and Phillip A. O'Hara (2011) focus on global money and financial architecture in the light of the last financial and economic crisis.

Theoretical models explaining the stockpile of foreign reserves, using the approach of the precautionary view, often describe emerging countries as financing long term investment with volatile short term foreign capital and building reserves to cushion the real output of the economy in the event of foreign capital outflow. The impact of a sudden stop (of foreign capital inflow) on the output varies from one model to another. For example, in Jeanne and Romain Rancière (2006) and in Jeanne

(2007) the sudden stop lasts one period, after which the output goes back to its long run growth path, whereas in Pablo Garcia and Claudio Soto (2004), in Aizenman and Lee (2007) and in Yin-Wong Cheung and Xingwang Qian (2009) the sudden stop forces a costly liquidation of the investment, thereby reducing the output. Other theoretical studies focus on the effect of capital outflows on the real exchange rate (Avner Bar-Ilan and Marion 2009) take the target of reserves as given and neglect the financial account of the balance of payments; Timothly Kehoe and Kim J. Ruhl (2009) do not focus on the precautionary role of foreign bond holdings) and on the determination of capital outflows as solutions of a general equilibrium framework (Laura Alfaro and Fabio Kanczuk 2009; Devereux and Sutherland 2009).

This study derives the reserves to short term foreign debt ratio as an optimal choice of a Central Bank which acts in the interest of a representative firm. The firm, in turn, simultaneously chooses investment and long term finance, in a setting where renewal of foreign short term debt is subject to uncertainty. Short term finance, which depends on foreign capital inflow and domestic monetary policy, is volatile but cheaper, whereas long term private finance is more costly and the cost increases with quantity. The outcome of the model is that the firm maximising its expected profit is not concerned with hedging against short term finance fluctuations, but it is rather concerned with hedging against the risk of rising long term costly finance in connection with its investment opportunity. As a consequence, from the viewpoint of the Central Bank, building a stock of reserves equal to the short term foreign debt (as the Greenspan-Guidotti rule would imply) is not sufficient. Optimal hedging policies imply higher ratios.

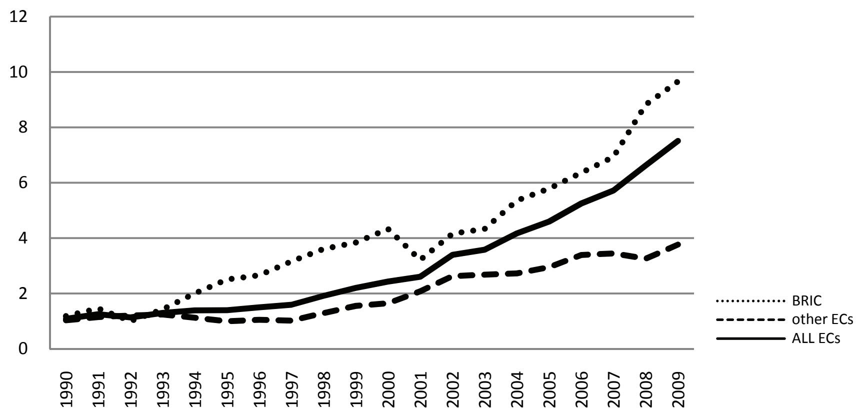
An approximated analytical solution of the model is derived to carry out a thorough sensitivity analysis of the determinants of the optimal reserves to short term foreign debt ratio and of its effects on the investment and financial structure. The optimal solution depends crucially on two structural parameters newly introduced in this model and representative of the openness of the emerging countries to global markets: they account for the marginal cost of long term finance and for the competitiveness of the domestic industry. Numerical simulations of the model reproduce the rising level of reserves in connection with rising GDP growth and rising long term finance as a result of simultaneous changes in both aforementioned openness parameters and derive almost half of the observed rise in GDP growth during the decade 2000-09 as a pure effect of the decreasing cost of long term finance.

The remainder of the paper is organised as follows. In Section 1, stylised facts are inferred from a descriptive analysis of the emerging countries. In Section 2, the model is presented and its approximated analytical solution is derived. Section 3 comments on the properties of the optimal solution for reserves to short term foreign debt ratio and its effects on investment and long term finance and derives numerical solutions compatible with observed stylised facts. Section 4 concludes.

1. Evolution of Emerging Countries over Time

The empirical descriptive analysis of this section is based on a sample of 18 emerging countries distributed across Asia, Latin America and Africa from 1990 to 2009. The sample is split into two subsamples: the BRIC economies (Brazil, Russia, India and China), which in the last decade account for more than 60% of the sample's GDP, and the other 14 smaller economies (less than 40% of the sample's GDP). Due to policy coordination, South Africa nowadays is often considered a new member of the group of strongest emerging (BRICS) economies. In this paper, however, it is considered more homogeneous with the smaller emerging countries as its GDP accounts for about 2.6% of the sample.

Figures in this section report the evolution of variables for the weighted average of the two subsamples and the full sample.

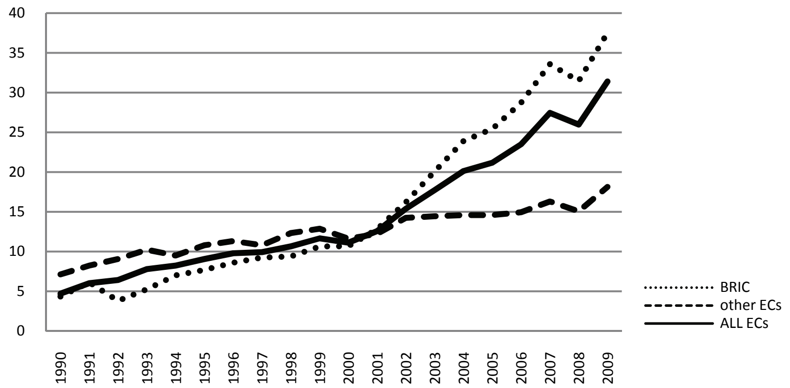


Source: World Bank dataset. Author's calculation.

Figure 1 Reserves to Short Term Foreign Debt Ratio

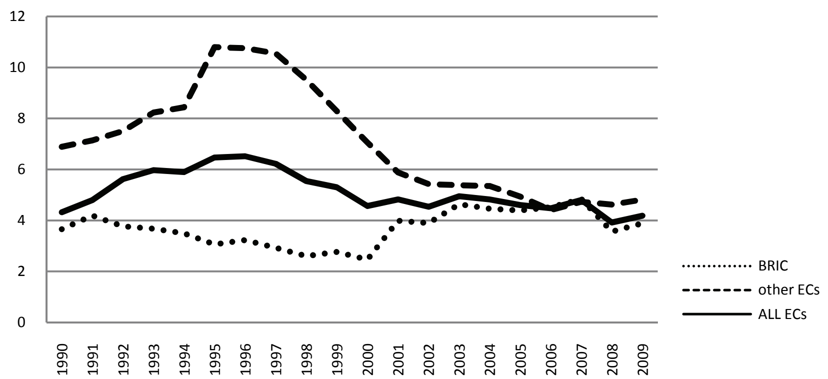
As Figure 1 illustrates, the ratio of reserves to short term foreign debt climbed from around 1 in 1990 to around 7.5 in 2009. The increasing level of this ratio was particularly pronounced in the group of bigger economies. Figure 2 indicates that the rise in the reserves to short term foreign debt ratio is associated to the increasing demand for international reserves. This was around 5% of GDP in 1990 and rose constantly to around 30% in 2009, with the exception of difficult years (2000, 2008), when reserves were partially reduced. By contrast, the full sample short term foreign debt (Figure 3) presents a relatively stable path between 4% and 5% of GDP, suggesting that on average the rising values of the aforementioned reserves to short term ratio can be explained more by the increasing demand for foreign reserves than by the decreasing level of the short term foreign debt. Deleveraging contributed more intensively to raising the ratio in the smaller economies during the four years following the Asian Crisis.

The stylised fact that the demand for reserves continued to increase even during periods when the short term foreign debt decreased suggests that the Greenspan-Guidotti rule is not perceived as a sufficient recommendation to prevent new crises and that the short term foreign debt is not the only variable to take into account when the foreign reserve policy is decided.



Source: World Bank dataset. Author's calculation.

Figure 2 Foreign Reserves to GDP (Percent Values)



Source: World bank dataset. Author's calculation.

Figure 3 Short Term Debt to GDP (Percent Values)

The precautionary view emphasises that short term foreign debt is used up by emerging countries to finance long term investment projects (Roberto Chang and Andres Velasco 2001; Jeanne and Rancière 2006; Aizenman and Lee 2007; Jeanne 2007). Assessing the overall effect of short term foreign debt on the level of investment is a difficult task. In our sample, merely comparing the average values in the years 1990-99 and 2000-09, reported in Table 1, both the net investment (defined as gross fixed capital formation less depreciation) and the short term foreign debt decreased in the smaller economies and increased for the BRIC countries. From a closer look at the trends, it can be observed that the ratio of short term foreign debt to net investment (Figure 4) after 2000 was stable in BRIC economies and fell in the smaller countries.

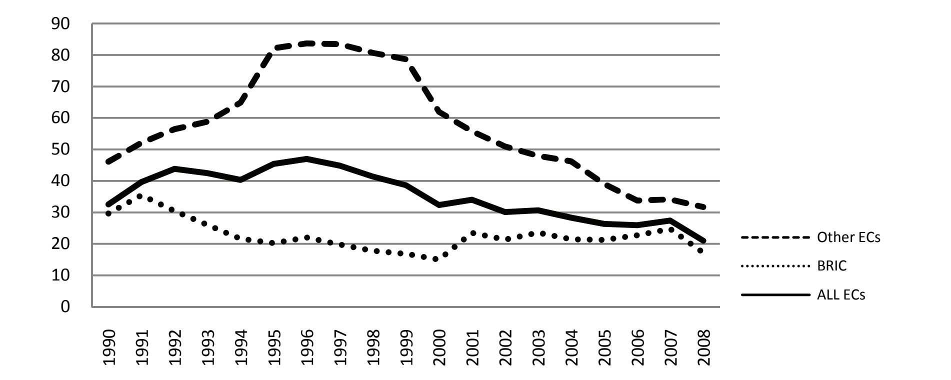
While the short term foreign debt seems to play a diminishing role, the relevance of other sources of longer term finance (foreign and domestic) is clearly increasing. Table 1 reports the average values of three sources: foreign debt with maturity over one year, international flows in the capital market (to finance bonds and new equity issues) and the volume of the stock traded. A proxy for long term finance (LTF) is built on the reported sources of finance, based on the simplifying assumption that the ratio between short and long term finance in the capital market is the same as for the foreign debt. LTF proxy to GDP over time (Figure 5) was considerably higher in the group of smaller economies until 2006 (except for the year 2000), and was quite volatile in the BRIC group, where it rose by 87% on average in the second decade (Table 1).

Table 1 Evolution of Emerging Economies: Growth, Investment, Short Term Debt and Reserves

1990-99	2y growth	NI/ GDP	STD/ GDP	RES/ GDP	RES/ STD	LTD/ GDP	IKM/ GDP	ST/ GDP	LTF/ GDP
All ECs (av.)	8.36	13.56	7.31	11.62	1.59	35.51	1.93	16.72	49.93
All ECs (w. av.)	9.26	13.60	5.67	8.41	1.484	25.69	1.73	15.13	39.50
BRIC	9.23	14.11	3.09	6.87	2.226	20.66	0.94	14.53	34.12
Smaller ECs	9.10	12.99	8.81	10.23	1.161	33.51	2.70	16.46	48.68
Argentina	9.41	5.59	7.66	6.48	0.846	28.87	3.33	3.90	34.59
Brazil	5.11	7.00	5.17	5.55	1.075	23.01	1.17	12.20	33.92
Chile	13.43	10.73	7.15	21.57	3.018	34.43	2.70	7.83	43.15
China	22.44	22.92	2.77	11.41	4.113	13.29	1.00	18.86	29.72
Colombia	5.32	8.19	5.34	11.60	2.172	28.72	1.75	1.26	31.26
Egypt, Arab Rep.	9.04	12.13	5.38	22.95	4.267	52.43	0.47	3.11	55.67
Indonesia	12.78	11.30	13.39	10.76	0.804	59.95	2.63	8.75	69.26
India	11.94	12.73	1.68	5.64	3.350	25.51	0.71	20.73	45.63
Morocco	4.45	25.96	2.66	12.62	4.744	70.59	0.68	2.66	73.81
Mexico	6.74	12.41	7.83	5.59	0.714	28.46	3.43	11.57	40.23
Malaysia	14.56	7.77	8.68	30.83	3.552	33.33	3.43	99.69	115.14
Pakistan	7.90	26.78	5.37	3.46	0.643	42.69	1.23	9.49	52.21
Peru	8.66	8.66	13.53	14.33	1.059	43.39	0.69	4.39	47.26
Philippines	6.35	11.89	9.55	11.82	1.238	54.60	3.39	16.44	71.48
Russian Federation	-8.13	10.84	2.78	2.79	1.004	26.94	1.02	1.71	29.42
Thailand	9.18	9.20	19.72	21.16	1.073	35.88	2.81	37.48	61.88
Turkey	7.53	30.05	8.48	7.10	0.837	31.29	2.77	18.39	47.94
South Africa	3.80	9.91	4.36	3.04	0.696	5.84	1.53	19.99	18.17
2000-09	**2y growth**	**NI/ GDP**	**STD/ GDP**	**RES/ GDP**	**RES/ STD**	**LTD/ GDP**	**IKM/ GDP**	**ST/ GDP**	**LTF/ GDP**
All ECs (av.)	11.31	11.7	5.26	19.22	3.657	29.89	2.79	31.76	56.79
All ECs(w. av.)	14.15	16.39	4.57	20.64	4.516	20.66	2.51	47.07	61.25
BRIC	16.63	19.22	4.07	24.04	5.912	15.06	2.24	59.73	63.85
Smaller ECs	10.15	12.16	5.26	14.62	2.779	29.44	2.88	24.71	52.84
Argentina	8.29	7.38	11.85	12.61	1.065	55.36	1.39	3.30	59.22
Brazil	7.06	5.59	3.50	9.02	2.574	24.05	3.36	23.15	47.19
Chile	8.04	8.85	8.03	16.87	2.100	37.73	4.29	14.68	53.37
China	22.01	28.68	4.90	32.26	6.588	6.77	1.23	79.36	53.53
Colombia	8.43	8.20	3.17	10.36	3.267	25.55	2.37	2.99	30.32
Egypt, Arab Rep.	9.98	12.12	2.15	19.37	9.004	27.27	2.25	21.31	49.10
Indonesia	32.30	8.74	6.82	14.06	2.063	41.91	1.73	13.56	55.05
India	14.98	18.11	1.77	16.95	9.599	16.63	1.64	68.04	79.62
Morocco	10.05	19.03	2.63	26.35	10.012	32.11	0.58	13.08	44.73
Mexico	4.29	17.06	1.87	8.34	4.467	20.21	2.80	7.21	29.38
Malaysia	10.10	9.00	8.95	44.00	4.918	30.76	6.04	44.19	69.67
Pakistan	9.69	13.93	1.42	9.11	6.407	32.69	0.57	59.83	90.58
Peru	11.29	6.84	4.57	19.72	4.315	33.83	1.89	2.97	38.11
Philippines	9.58	9.71	5.88	21.48	3.654	55.35	6.49	8.47	68.87
Russian Federation	12.30	7.64	4.80	22.87	4.767	30.85	4.57	28.67	59.61
Thailand	8.96	6.70	9.37	33.68	3.596	24.89	1.38	45.98	59.31
Turkey	8.37	16.31	7.36	10.93	1.485	31.56	3.60	41.14	67.84
South Africa	7.97	6.81	5.56	8.68	1.561	10.60	3.99	93.82	74.77

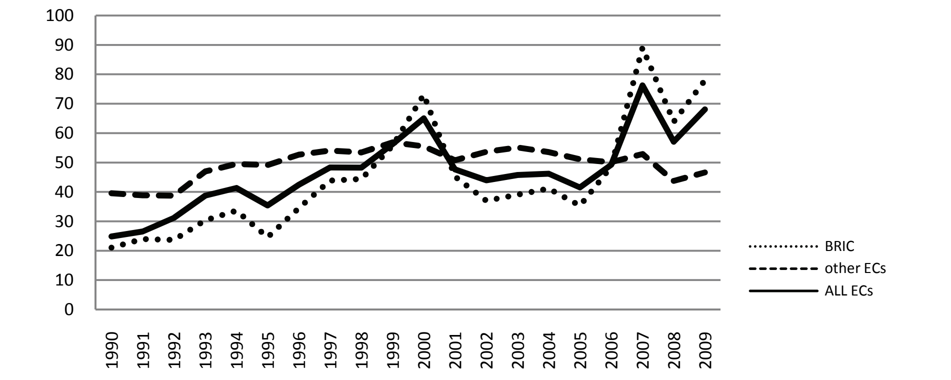
Note: Average values of 2 years GDP growth (2y growth), net (start up) investment in fixed capital assets over GDP (NI/GDP), stock of short term foreign debt over GDP (STD/GDP), stock of foreign reserves over GDP (RES/GDP), reserves to short term foreign debt ratio (RES/STD), stock of long term foreign debt with maturity higher than one year over GDP (LTD/GDP), financing via international capital market over GDP (IKM/GDP), stock traded over GDP (ST/GDP), proxy for long term finance to GDP (LTF/GDP). NI is defined as gross fixed capital formation less depreciation; LTF/GDP is computing by summing all sources of finance (STD+LTD+IKM+ST) and multiplying by the ratio of long term over total foreign debt. Values refer to two different time periods (1990-99 and 2000-09) for each single country of the sample and for the following groups: Full sample (average and weighted average values), BRIC economies (weighted average values) and smaller economies (weighted average values).

Source: World Bank dataset. Author's calculation.



Source: World Bank dataset. Author's calculation.

Figure 4 Short-Term Foreign Debt to Net Investment (Percent Values)

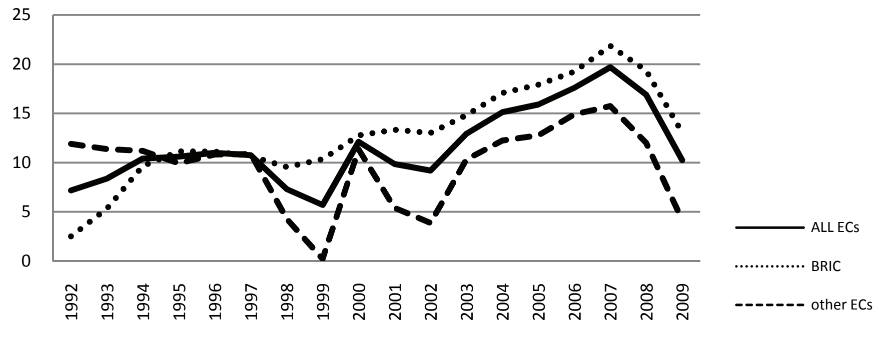


Source: World Bank dataset. Author's calculation.

Figure 5 Long Term Finance to GDP (Percent Value)

Figure 6 illustrates how the long term GDP growth (average over 2 years) increased sharply for 5 years from 2002 to 2007, to an extent that the growth in the net investment is not able to explain. The average values reported in Table 1 confirm this result: 2 year growth rates increased by 80% in the BRIC countries while the net investment increased by less than 4%, and increased in the smaller countries even though the net investment decreased.

Overall, the empirical descriptive analysis of this section allows us to establish the following stylised facts: firstly, the stock of reserves grows independently from short term foreign debt; secondly, there is no clear relation between short term foreign debt and investment in new capital assets; thirdly, countries have been able to grow considerably in the last decade even though investment in new capital assets has not changed to a relevant extent; fourthly, massive reserves accumulation, rather than causing underinvestment and slowing down the economy, seems to have grown, as has GDP.



Source: World Bank dataset. Author's calculation.

Figure 6 Growth of GDP over 2 Years (Percent Values)

While the evidence presented in this section seems to contrast with existing explanations of the rationale for (and consequence of) reserve accumulation, the next section finds a theoretical explanation which makes all the aforementioned stylised facts consistent with each other.

2. Reserves, Investment and Finance

This section builds a partial equilibrium theoretical model describing the optimal solutions for a representative firm, which determines the supply of goods, and a Central Bank, which determines the optimal stock of foreign reserves, in a setting where renewal of foreign short term debt is subject to uncertainty.

The model describes a partial equilibrium in that it takes as given the demand for produced goods from both the domestic consumer and the foreign sector and does not consider the trade balance and the exchange rate. All variables are denominated in domestic currency. The assumption of constant exchange rate is not needed, as the quantities involved (foreign reserves, investment, long term finance, final output), determined as endogenous solutions, are able to capture price effects. In this respect, this study differs from most of the models belonging to the precautionary view, surveyed in the introduction, which associate capital reversal to output drop while leaving unchanged (Jeanne and Ranciere 2006; Jeanne 2007) or unspecified[1] the value of the other national quantities.

[1] In Aizenman and Lee (2007), bank liabilities come from foreign saving only. Other models, such as Garcia and Soto (2004), Caballero and Panageas (2004, 2005), do not commit themselves to explicit relations between national aggregate quantities, and adopt generic variables to be measured empirically, such as "cost of a crisis" or "need for funds during the sudden stop."

Subsection 2.1 derives a non closed form solution, subsection 2.2 an analytical approximated solution. The latter is used in Section 3 for a thorough sensitivity analysis and for carrying out the numerical simulations.

2.1 Optimal Solutions for a Firm and a Central Bank

For simplicity, we assume that all firms are identical and that population size is equal to 1. Therefore the model describes a single firm but all variables involved denote national aggregate quantities. As in previous studies on the role of reserves, we assume that liquidity shock may force underinvestment, reducing second period output. As our focus is on developing countries, we assume that domestic long term investment of the firm is financed by: (i) *cash flow* from preexisting assets; (ii) *short term finance* from banks; (iii) *long term finance*, from any sources (long term debt from banks, bonds, new equities, capital venture, etc.). Short term finance depends on two components: supply of foreign short term credit and supply of domestic credit. We assume that domestic credit is cheaper but scarce (for simplicity, interest rate is zero), whereas the cost of foreign short term credit is higher and constant ($r > 0$). We also set the interest rate on foreign reserves as equal to zero, therefore r also represents the spread between the low yield on liquid reserve assets and the cost of external borrowing (Dean Baker and Karl Walentin 2001; Dani Rodrik 2006; Stiglitz 2006). Demand for short term credit to finance the investment is perfectly elastic, as the long term finance is more expensive. We assume that raising long term finance is costly and the cost increases with the quantity.

The time line is summarised in Table 2. Investment starts at time 0 with a new fixed (start up) capital factor, K, and is completed at time 1 with a variable component, I. The variable investment I includes payment to all factors which are different from the initial fixed capital (intangible capital, human capital, workers, capital replacement, etc.).

Table 2 The Time Line

	Time 0	Time 1	Time 2
EXOGENOUS VARIABLES	Investment in net capital assets (K) raising funds from short term foreign debt (D_0) and pre-existing sources (V).	Shock to short term foreign capital (ε).	
ENDOGENOUS VARIABLES		Short term finance available (D_1) determined by ε and h^*.	Return on output. Service of finance. Net profit π.
DECISIONS	Central bank's reserves policy h^*.	Firm's variable investment and long term finance I^*, B^*.	

Source: Author's model.

Physical output is realised at time 2 and given by

$$f(K, I) = \omega K^{\alpha} I^{\beta}. \tag{1}$$

We adopt the conventional (but unnecessary) assumption of constant return to scale: $\alpha + \beta = 1$. For simplicity, we assume that at time 0 the only source of short term finance is from foreigners, and the amount is given by D_0. Time 0's start up capital is thus

$$K = D_0 + V, \tag{2}$$

where V is cash flow from preexisting assets. Time 1's investment is financed with (domestic and foreign) short term finance, D_1, and long term finance, B:

$$I = D_1 + B. \tag{3}$$

At time 1, the firm has to service the previous period's debt (principal and interests) and renewal of foreign debt is hit by a multiplicative shock ε, distributed as a normal $N(1, \sigma^2)$; therefore, available funds from foreigners are given by $D_0(\varepsilon - 1 - r)$; the firm at time 1 also raises short term debt from domestic credit, ΔDC. The latter is determined by the monetary policy. Recalling that change in money supply is the sum of changes in domestic credit and foreign reserves, i.e. $\Delta M = \Delta DC + \Delta R$, we assume that the central bank at time 1 fulfills the following simple rules:

1. *Precautionary policy*: the proportion of foreign reserves to private foreign short term capital inflow is constant (h) if the net capital inflow is positive ($\varepsilon > 0$), zero otherwise;

2. *Sterilisation of foreign currency operations*: if the net capital inflow is non negative at time 1, money supply is unchanged ($\Delta M = 0$ if $\varepsilon \geq 0$);

3. *Expansionary policy during a crisis*: if the net capital inflow is negative at time 1, new money is issued in proportion h of the net capital outflow ($\Delta M = -hD_0\varepsilon$ if $\varepsilon < 0$).

From the aforementioned rules, reserves at times 0 and 1 are given, respectively, by

$$R_0 = hD_0 \tag{4}$$

and

$$R_1 = \begin{Bmatrix} hD_0\varepsilon & \text{if } \varepsilon \geq 0 \\ 0 & \text{if } \varepsilon < 0 \end{Bmatrix}, \tag{5}$$

whereas domestic credit at time 1 is given by

$$\Delta DC = -hD_0(\varepsilon - 1). \tag{6}$$

Summing up domestic and foreign credit yields the total short term finance available to the firm at time 1:

$$D_1 = D_0[h + (1 - h)\varepsilon - (1 + r)]. \tag{7}$$

At time 2 the output is sold at price θ and revenues are given by $F(K, I) = \theta f(K, I)$. We make the assumption that a negative relation (even very small) exists between time 2's output price and time 1's capital inflow:

$$\theta = \eta(\varepsilon - 1) + 1 \tag{8}$$

with $\eta < 0$. The coefficient η captures the intensity of this relation and accounts for the competitiveness of the product's industry in this small open economy.

The more the industry is exposed to local and global competition, the more any additional foreign capital inflow is likely to be related to a downward shift in the supply curve. Marginal cost reduction could be due to the birth of new firms, to lower market power of previous existing companies, to lower power of trade unions. More competition, thus, leads to a lower final output price.

The cost of long term finance is a growing function of the total amount. To keep the analysis as simple as possible, we model this cost as an exponential function:

$$C(B) = sB^{1+\delta}, \tag{9}$$

where s is a scale parameter and the coefficient δ accounts for the cost that the firm has to pay to increase long term finance. δ takes positive values and is expected to be lower in more globalised emerging economies with a more developed financial market.

The firm enters time 1 with the given stock of capital K and with available short term finance D_1 and chooses investment I (and thereby the amount of long term finance B) to maximise net expected profits (assuming discount rate equal to 1 for simplicity):

$$\pi = \theta f(K, I) - I - D_0 \varepsilon r - C(B). \tag{10}$$

At time 1, D_1 is given, $\frac{dB}{dI} = 1$, hence the first order condition for this problem is:

$$\theta f_I = 1 + C_B, \tag{11}$$

where f_I and C_B are the first derivatives of (1) and (9), respectively, with respect to I and B.

The Central Bank acts in the general interest of the economy and chooses optimal h by maximising the expected profit of the firm, subject to available information at time 0, when the future foreign capital inflow is still uncertain:

$$\max_h E_0 \pi\big(D_1(\varepsilon, h)\big). \tag{12}$$

Based on the result of Kenneth A. Froot, David S. Scharfstein, and Jeremy C. Stein (1993), the non-closed form solution to (12) is given by the following formula:

$$h^* = 1 + \frac{\eta}{D_0} \frac{E_0\left[\dfrac{-f_I C_{BB}}{\theta f_{II} - C_{BB}}\right]}{E_0\left[\dfrac{-\theta f_{II} C_{BB}}{\theta f_{II} - C_{BB}}\right]}, \tag{13}$$

where f_{II} and C_{BB} are second derivatives of (1) and (9).

The informational content of the formula for the optimal solution (13) does not allow one to derive easily evident implications on the relationship between parameters involved in the reserve policy decision. Expression (13) shows clearly that the reserve policy depends on the parameter η, capturing the relation between return on investment and foreign capital fluctuations. The closer to zero η is, the lower the reserves to short term foreign debt ratio, h, which is equal to 1 (the value of the

Greenspan-Guidotti rule) when there is no relation between capital inflow and return on investment. Clearly, the decision on reserves is also dependent on the second moment of the shock to capital inflow (ε), which is included in the second derivatives f_{II} and C_{DD}. However, (13) does not show exactly how the volatility of the shock (σ), the competitiveness parameter (η) and the short term foreign credit available at the date when the decision on reserves is taken (D_0) affect the decision on optimal reserves. Moreover, as reserves in this model are a tool to coordinate financing and investing policy, more information would also be desirable on how reserves depend on the parameters affecting investment and cost function (and the concavity of the payoff function), as well as on how investment, short term and long term finance are in turn affected by reserve policy. The next subsection makes this issue clearer.

2.2 Locally Approximated Solution

Equations (3), (11) and (13) constitute an unsolved system of three equations with three unknowns: I, B, h. Solving this system of equations would lead to expressing the three unknowns as functions of the random variable, ε. The system can be solved as a local approximation, after a second order Taylor expansion of the investment and equity cost functions, (1) and (9) respectively, around the expected levels of the investment, $\bar{I}$, and equity, $\bar{B}$.

After the second order Taylor expansion, the expected revenue and cost functions defined above take the following quadratic forms:

$$f(I) = \frac{a}{2}I^2 + bI + k, \tag{14}$$

with $a = f_{II}(\bar{I}) < 0$, $b = f_I(\bar{I}) - \bar{I}f_{II}(\bar{I}) > 0$ and $k = f(\bar{I}) - \bar{I}f_I(\bar{I}) + \frac{1}{2}\bar{I}^2 f_{II}(\bar{I})$, where $f_I = aI + b$, $f_{II} = a$;

$$C(B) = \frac{c}{2}B^2 + dB + w, \tag{15}$$

with $c = C_{BB}(\bar{B}) > 0$, $d = C_B(\bar{B}) - \bar{B}C_{BB}(\bar{B}) > 0$ and $w = C(\bar{B}) - \bar{B}C_B(\bar{B}) + \frac{1}{2}\bar{B}^2 C_{BB}(\bar{B})$, where $C_B = d + cB$ and $C_{BB} = c$.

Substituting (14) and (15) into the expected profit function (10), time 1's f.o.c. simplifies to

$$\theta(aI + b) = 1 + d + cB. \tag{16}$$

Combining (16) with time 1's budget constraint (from (3) and (7)),

$$I = B + D_0[h + (1-h)\varepsilon - (1+r)],$$

we can derive the optimal investment and long term finance as functions of the shock to capital inflow, ε, and the optimal reserves to debt ratio, h:

$$I^*(h, \varepsilon) = \frac{(1+d) - \theta b - cD_0[h + (1-h)\varepsilon - (1+r)]}{(\theta a - c)} \tag{17}$$

$$B^*(h, \varepsilon) = \frac{(1 + d) - \theta b - \theta a D_0[h + (1 - h)\varepsilon - (1 + r)]}{(\theta a - c)}. \tag{18}$$

From a second order Taylor expansion of the two expected terms of equation (13) around $\varepsilon = 1$, after substituting for the approximated functions' derivatives, f_I, f_{II}, and C_{DD}, and for the optimal investment (equation (17)) into the expression for f_I, one can derive the reserves to short term foreign debt ratio:

$$h^* = 1 + \frac{\eta}{D_0} \frac{[1 + d - \frac{bc}{a} + cD_0 r][(a - c)^2 + 3a^2\eta^2\sigma^2]}{(a - c)[(a - c)^2 + 3ac\eta^2\sigma^2]}. \tag{19}$$

Expression (19), unlike expression (13), displays the exact relationships between the parameters involved in the determination of the optimal ratio. Substituting expression (19) into equations (17) and (18) yields the analytical solutions for investment and long term finance levels as functions of the shock to capital inflow, ε.

3. Implications of the Model

With the approximated analytical solution it is possible to derive, firstly, some propositions about the determinants of the optimal ratio of reserves to short term foreign debt, secondly, the effects of reserves accumulation on the firm's investment and finance decisions and, thirdly, the effects on all variables of some changes in the parameters accounting for openness to global markets. To implement this sensitivity analysis, numerical simulations of the theoretical model are carried out to mimic observed average data reported in Table 1. These should not be interpreted as a tentative empirical confirmation of the model's propositions, but as exercises to identify and quantify the relationship between variables involved in the partial equilibrium framework adopted.

Figures throughout this section refer to the full sample's weighted average values only, whereas Table 4 in subsection 3.3 also refers to weighted average values of the subsamples (BRIC and smaller countries). In calibrating the model, we take as given time 0 data, i.e. the values of the fixed investment ($K = NI/GDP$) and short term foreign debt ($D_0 = STD/GDP$), and we infer the values of the parameters η, σ and δ compatible with observed long period (2 years) growth, reserves to GDP ratio and proxy for long term finance. We assume that the elasticity and scale parameters of the investment function are fixed and take standard values, although the factors involved (K, I) are not defined in a standard way. The fixed capital share (NI in Table 1) is, thus, $\alpha = 0.25$ and the variable investment's share is $\beta = 0.75$. The value of the interest rate on foreign short term debt (r) is computed from the data available by averaging the interest payment divided by the stock of short term foreign debt; it is set equal to 5% in 1990-99 and to 4% in 2000-09. The cost of long term finance function, by contrast, is inferred from simulations, as there are no data available and the heterogeneity of the sources involved in the aggregate proxy for long term finance does not allow the cost to be measured convincingly. The scale parameter multiplying the cost of long term finance function ($s = 0373$) is set constant throughout all numerical simulations, in order to focus on marginal cost changes only (δ). This setting is consistent with values of the average cost of long term finance (presented in Table 4) which are higher than the short term interest rate and vary from a maximum of nearly 18% (BRIC countries in 1990-99) to a minimum of nearly 6% (smaller countries in 2000-09). The parameter multiplying the investment function (ω), accounting for total factor productivity, is allowed to change to derive the residual part of long period average growth which is not attained by modifying the model's structural parameters associated to globalisation.

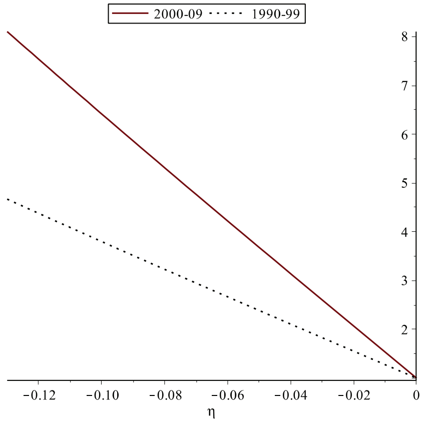
3.1 Optimal Reserves to Short Term Foreign Debt

This subsection derives and discusses some properties of the optimal reserves to short term foreign debt ratio implied by the approximated analytical solution (proofs of the propositions are in the appendix).

Proposition 1 *The optimal reserves to short term foreign debt ratio, h^*, is a decreasing function of the parameter η for any σ^2 lower than the critical value $\sigma^{*2} = \frac{-(a-c)^2}{3\eta^2 ac}$.*

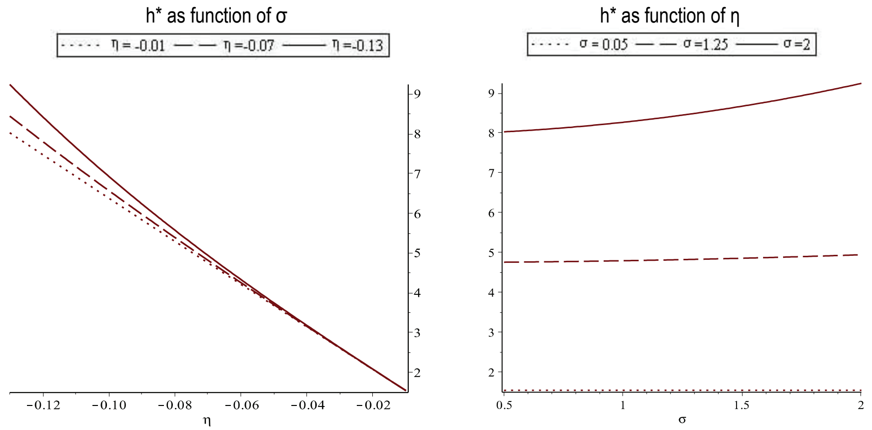
This proposition confirms the general result (equation (13)) that the higher the correlation between return on investment and foreign capital inflow, the lower the optimal ratio h^*, but it also adds a limit: if the volatility of the foreign capital inflow is too high, no clear monotonic relation can be computed in the locally approximated solution between ratio h^* and competitiveness parameter η. The maximum critical value of the variance, σ^{*2}, depends on the concavity of the profit function, expressed by the parameters a and c, and on the absolute value of the competitiveness parameter, η. With the very small values of the competitiveness parameter examined in this section, the upper bound volatility is virtually infinite, thus the decreasing relation is always verified.

Figure 7 illustrates the aforementioned relation for our sample, based on 1990-99 (dashed line) and on 2000-09 data (continuous line). The higher sensitivity of the optimal ratio for the 2000-09 period depends mainly on the lower value assigned to the marginal cost of the long term finance (i.e. lower δ, which affects the values of c and d in the approximated solution (19)) to be consistent with the average value of the long period growth (see subsection 3.3 and Tables 3 and 4 for a more detailed explanation). In both lines, the closer to zero the value of η is, the closer the optimal ratio h^* is to the Greenspan-Guidotti rule. When the relation between return on investment and foreign short term debt is negative, the policy of offsetting outflows of short term foreign credit with newly generated domestic credit (by selling reserves) is not sufficient, as the investment needs to raise more finance. Ratios higher than one, therefore, are needed to prevent the firm from raising an extra amount of longer term finance. Generally speaking, *the firm maximising its expected profit from a concave profit function is not concerned with hedging against the short term finance fluctuations, but it is rather concerned with hedging against the risk of raising long term finance in connection with its investment opportunity*.



Source: Author's simulation based on weighted average values reported in Table 1.

Figure 7 h* as function of η



Source: Autor's simulation based on weighted average values of the sample during years 2000-09 reported in Table 1 (δ=0.2415, D_0=4.57, K=16.39).

Figure 8 Irrelevance of the Variance (σ)

Proposition 2 *Provided that $\sigma^2 < \sigma^{*2}$, given any $\eta < 0$, the higher the variance of the shock to short term foreign debt, σ^2, the higher the reserves to short term foreign debt ratio, h^*.*

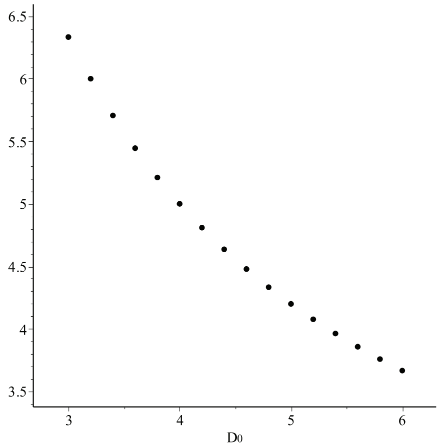
This proposition is illustrated in Figure 8, where h^* is expressed either as a function of the volatility parameter, σ, setting three different levels of η (-0.01, -0.07, -0.13), or as a function of the competitiveness parameter, η, setting three different values of σ (0.5, 1.25, 2). Clearly, the closer the value of η is to zero, the lower the sensitivity of h^* is to rising volatility. As the values of η considered in the context of the emerging economies are very low, a change in volatility has a negligible impact on the choice of reserves, variable investment, and long term finance.

The irrelevance of the variance is not an intuitive result: more volatile short term foreign capital inflows have generated worries and justified raising reserves with a precautionary motive. However, according to this model, in line with Radelet and Sachs (1998) and Rodrik (2006), the fact that foreign short term capital is volatile justifies the one-to-one ratio suggested by the Greenspan-Guidotti rule only, but does not explain rationally the higher ratios observed in recent years.

Proposition 3 *For any $\eta < 0$, the optimal ratio h^* is a decreasing function of the short term foreign debt, D_0.*

This is another counterintuitive result: common wisdom (and the Greenspan-Guidotti rule) associates more foreign debt to more reserves. Evidence reported in Section 1, however, suggests that this link is not confirmed and that reserves grow independently from short term foreign debt. Proposition 3 allows us to explain the aforementioned evidence: the simple presence of short term foreign debt in a country justifies an equal quantity of reserves, but not more. This model, by contrast, deals with situations where countries hold higher ratios as they need to hedge against long term costly finance. When short term finance rises, the gap between investment and long term finance becomes lower, as does the marginal cost of long term finance. Hence, the incentive to substitute extra short term finance with extra long term finance is lower and the reserves to short term foreign debt ratio approaches the level recommended by the Greenspan-Guidotti rule.

Figure 9 illustrates this relationship by simulating optimal solutions of the model for different values of D_0, *ceteris paribus*. The figure reports single point solutions for h^*, as for every value of D_0 all parameters of the local approximated analytical solution around the expected investment ($\bar{I}$) and long term finance ($\bar{B}$) are recalculated.



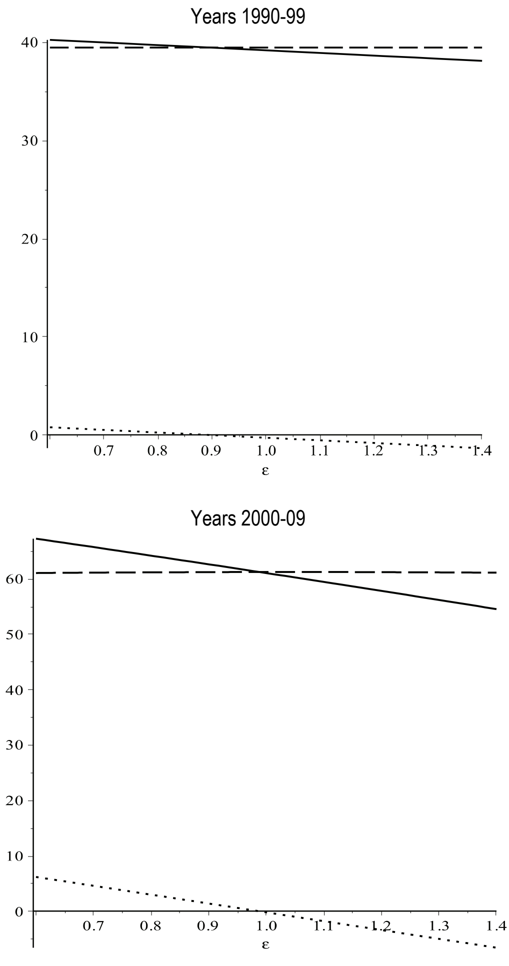
Source: Author's simulations based on weighted average values of the sample during years 2000-09 reported in Table 1 (η=-0.065; σ=0.8; δ=0.2415, K=16.39).

Figure 9 h* as Function of D_0

3.2 Effects of the Shock to Short Term Foreign Debt

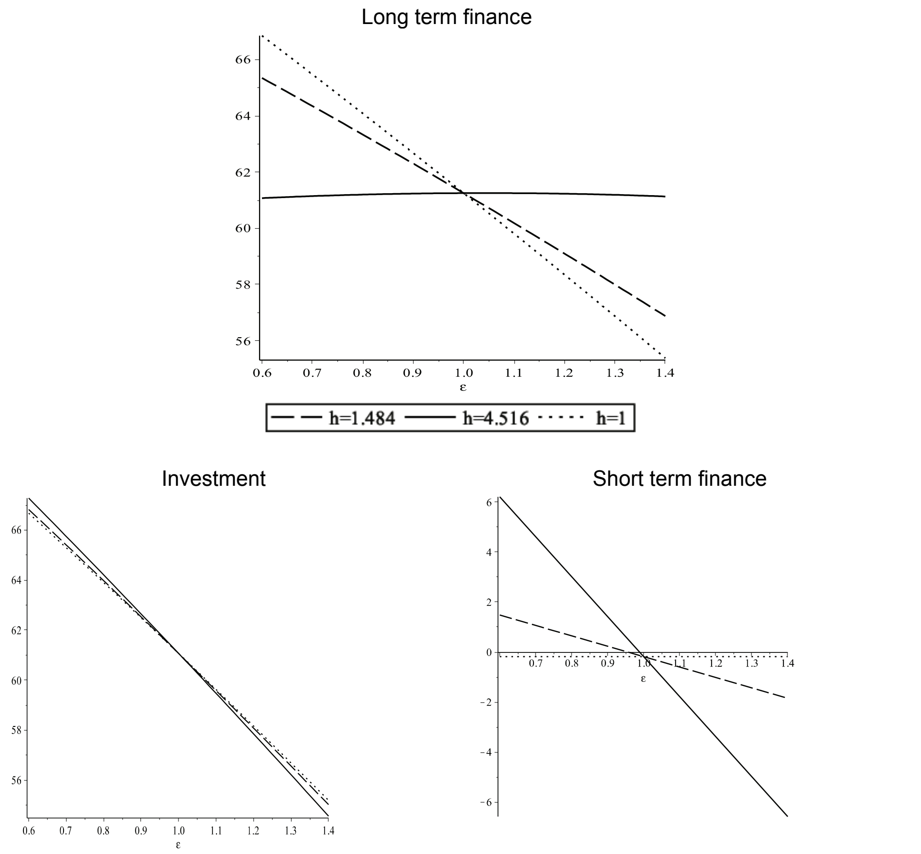
Based on the optimal ratio (19), this subsection discusses the model's implications about the effect of a shock to short term foreign debt on optimal decisions about investment (17) and long term finance (18), as well as on the short term finance available to the firm (7).

Figure 10 illustrates two examples taken from weighted average values of the emerging countries in the two decades examined in Section 1. The values expected at time zero (i.e. the points corresponding to $\varepsilon = 1$) are also reported in Table 4, Panel A, first and last columns. The figure illustrates how reserve policy coordinates investment, short term and long term finance.



Source: Author's simulations based on weighted average values of the sample reported in Table 1.

Figure 10 Investment, Short Term Finance and Long Term Finance as Functions of the Shock to Capital Inflow (Optimal Solutions)



Source: Author's simulations based on weighted average values of the sample during years 2000-09 reported in Table 1.

Figure 11 Investment, Short Term Finance and Long Term Finance with Different Reserve Ratios

As the return on investment is negatively related to the shock to foreign capital inflow ($\eta < 0$), the variable investment decreases with ε. The short term finance at time 1 is also a decreasing function of the shock ε, for reserves to short term ratios higher than one. More exactly, the higher h is, the higher the slope of the short term finance function is (it would be a flat line for $h = 1$ recommended by the Greenspan-Guidotti rule). The optimal ratio, therefore, makes it possible to generate short term finance when the foreign capital inflow slows down ($\varepsilon < 1$) and more investment is needed, and to reduce it when capital inflow increases ($\varepsilon > 1$) and investment slows down. The result of the optimal ratio is to fully stabilise (to a virtually flat line) the long term finance function around its expected level. The different levels of the expected variable investment (and therefore long term finance) at time 1 in the years 1990-99 and 2000-09 are due to changes in the cost of long term finance, as we will see in the next subsection. A lower cost of external finance is an incentive to increase the variable investment, given the fixed (start up) capital factor and the expected foreign capital inflow.

Figure 11 compares firm's investment, short term and long term finance in the period 2000-09, with three alternative values of the reserves to short term foreign debt ratio, h. The Greenspan-Guidotti rule ($h = 1$) implies full stabilisation of short term finance available to the firm, lower variability of the investment but much higher variability of the more expensive long term finance. The ratio of reserves to short term foreign debt that was optimal in the 1990-99 period ($h = 1.484$) would still imply a considerable variability of the long term finance in the 2000-09 period. By contrast, the optimal ratio in 2000-09 ($h^* = 4.516$), based on changed values of the parameters of industry (lower η) and the financial market (lower δ), generates higher fluctuations in the short term finance, higher sensitivity of the investment to foreign capital inflow, but nearly full stabilisation of the long term finance function. This result confirms, again, that the purpose of reserve policy is to stabilise not short term finance, but the longer term finance available to the investment. The Greenspan-Guidotti rule is not an optimal solution because it simply stabilises short term finance.

3.3 Effects of Globalisation

This subsection reports numerical simulations of the model presented in Section 2 with two objectives. The first one (Table 3, Figure 12) is to analyse how sensitive the solutions of the model are to the parameters associated to globalisation, namely, δ and η. The second one (Table 4) is to separate and isolate the effect of three determinants of GDP growth implied by the theoretical model: the change in the stock of fixed capital, the decreasing cost of long term finance and the increasing total factor productivity.

The simulations quantify the empirical implications of the theoretical model when the given variables take the value of the sample examined in Section 1 of this study. The solution of the model depends on three parameters whose changing value can be associated to the process of globalisation. They account for the marginal cost of long term finance, δ, the competitiveness of the domestic industry, η, and the volatility of the foreign short term capital, σ. As the volatility parameter is irrelevant for values of η that are sufficiently low (from proposition 2), this subsection examines the effects of changing values of δ and η. The purpose of the simulations is simply to focus on the role played by the structural parameters η and δ, within the partial equilibrium framework built in this work, to determine the stylised facts observed in Section 1. The numerical exercises should not be considered as appropriate estimates of the values of the two mentioned parameters: this could be the object of further work.

Table 3 refers to the full sample and computes solutions for the values of η from -0.01 to -0.13 and δ from 0.2 to 0.42. Each solution implies values for time 0's optimal reserves to short term foreign debt ratio and for time 1's expected variable investment, total investment, long term finance, average cost of long term finance, long period (2 years) growth. Bold characters are values of the 2 years growth, the long term finance proxy and the optimal ratio h^* compatible with those observed in our sample (Table 1).

Table 3 illustrates the fact that lower values of δ are not consistent, alone, with the observed increased ratios of reserves to short term foreign debt ratios, h^*. In principle, lowering the cost of long term finance should even induce a reduction in the stock of reserves, as it reduces the incentive to substitute extra short term finance with extra long term finance. Reserves, however, rise, because a lower value of δ also boosts the expected level of variable investment, which in turn requires a higher expected level of long term finance. The average ratio h^* for the full sample was 1.478 in 1990-99, which corresponds to δ between 0.34 and 0.36 and η between -0.01 and -0.02 in panel A. Panel B presents results with parameterisation of the same sample in the 2000-09 decade (see also Table 4). It can be verified that a level of δ equal to that inferred in Panel A would imply a small rise in ratio h^*. On the other hand, decreasing levels of δ from 0.36 to 0.24 holding η constant, albeit able to mimic the observed growth and long term finance, would imply a ratio h^* increasing to a value between 1.515 and 2.073 only. The observed value, however, is 4.516 and is consistent with η between -0.06 and -0.07.

Table 3 Optimal Solutions for Different Values of Parameters

Panel A Full Sample - Weighted Average - Years 1990-99

RES to STD: h* η δ	**0.2**	**0.22**	**0.24**	**0.26**	**0.28**	**0.3**	**0.32**	**0.34**	**0.36**	**0.38**	**0.4**	**0.42**	**0.44**
-0.01	1.38	1.36	1.35	1.34	1.33	1.31	1.30	1.29	1.27	1.26	1.25	1.24	1.23
-0.02	1.75	1.73	1.70	1.68	1.65	1.63	1.60	1.57	1.55	1.52	1.50	1.48	1.45
-0.03	2.13	2.09	2.06	2.02	1.98	1.94	1.90	1.86	1.82	1.79	1.75	1.71	1.68
-0.04	2.51	2.46	2.41	2.36	2.31	2.25	2.20	2.15	2.10	2.05	2.00	1.95	1.91
-0.05	2.89	2.83	2.76	2.70	2.63	2.57	2.50	2.44	2.37	2.31	2.25	2.19	2.14
-0.06	3.27	3.19	3.12	3.04	2.96	2.88	2.81	2.73	2.65	2.58	2.50	2.43	2.36
-0.07	3.65	3.57	3.48	3.39	3.29	3.20	3.11	3.02	2.93	2.84	2.76	2.67	2.59
-0.08	4.04	3.94	3.84	3.73	3.63	3.52	3.42	3.31	3.21	3.11	3.01	2.92	2.82
-0.09	4.43	4.31	4.20	4.08	3.96	3.84	3.72	3.61	3.49	3.38	3.27	3.16	3.06
-0.1	4.82	4.69	4.56	4.43	4.30	4.17	4.03	3.90	3.77	3.65	3.52	3.41	3.29
-0.11	5.21	5.07	4.93	4.78	4.64	4.49	4.35	4.20	4.06	3.92	3.78	3.65	3.52
-0.12	5.61	5.45	5.30	5.14	4.98	4.82	4.66	4.50	4.35	4.19	4.05	3.90	3.76
-0.13	6.01	5.84	5.67	5.50	5.33	5.15	4.98	4.81	4.64	4.47	4.31	4.15	4.00
TI	66.9	65.2	63.4	61.5	59.7	57.9	56.0	54.2	52.4	50.7	48.9	47.3	45.7
NI	13.60												
STD	5.67												
VI	53.4	51.6	49.8	48.0	46.2	44.3	42.5	40.7	38.9	37.1	35.4	33.7	32.1
LTF	53.6	51.9	50.1	48.3	46.4	44.6	42.8	40.9	39.1	37.4	35.7	34.0	32.4
ISTD	-0.28												
ACLTF	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23
GDP g	11.3	11.1	10.9	10.6	10.4	10.1	9.83	**9.52**	**9.20**	8.85	8.49	8.11	7.72

Panel B Full Sample - Weighted Average - Years 2000-09

RES to STD: h* η δ	**0.2**	**0.22**	**0.24**	**0.26**	**0.28**	**0.3**	**0.32**	**0.34**	**0.36**	**0.38**	**0.4**	**0.42**	**0.44**
-0.01	1.58	1.56	1.54	1.51	1.49	1.47	1.45	1.43	1.41	1.39	1.37	1.35	1.33
-0.02	2.16	2.11	2.07	2.03	1.99	1.94	1.90	1.86	1.82	1.78	1.74	1.70	1.67
-0.03	2.73	2.67	2.61	2.55	2.48	2.42	2.35	2.29	2.23	2.17	2.11	2.05	2.00
-0.04	3.32	3.23	3.15	3.06	2.98	2.89	2.81	2.72	2.64	2.56	2.48	2.41	2.33
-0.05	3.90	3.79	3.69	3.58	3.48	3.37	3.26	3.16	3.05	2.95	2.85	2.76	2.67
-0.06	4.48	4.36	4.23	4.10	3.97	3.85	3.72	3.59	3.47	3.34	3.23	3.11	3.00
-0.07	5.07	4.93	4.78	4.63	4.48	4.33	4.18	4.03	3.88	3.74	3.60	3.47	3.34
-0.08	5.66	5.49	5.33	5.15	4.98	4.81	4.64	4.47	4.30	4.14	3.98	3.83	3.68
-0.09	6.26	6.07	5.88	5.68	5.49	5.29	5.10	4.91	4.72	4.54	4.36	4.19	4.02

-0.1	6.85	6.65	6.43	6.22	6.00	5.78	5.56	5.35	5.14	4.94	4.74	4.55	4.36
-0.11	7.46	7.23	6.99	6.75	6.51	6.27	6.03	5.80	5.57	5.34	5.12	4.91	4.71
-0.12	8.07	7.81	7.56	7.30	7.03	6.77	6.51	6.25	6.00	5.75	5.51	5.28	5.05
-0.13	8.68	8.41	8.13	7.84	7.56	7.27	6.98	6.70	6.43	6.16	5.90	5.65	5.40
TI	82.4	80.0	77.6	75.2	72.8	70.3	67.9	65.5	63.1	60.8	58.6	56.5	54.4
NI	16.39												
STD	4.57												
VI	66.0	63.6	61.3	58.8	56.4	53.9	51.5	49.1	46.8	44.5	42.2	40.1	38.0
LTF	66.2	63.8	61.4	59.0	56.6	54.1	51.7	49.3	46.9	44.6	42.4	40.3	38.2
ISTD	-0.18												
ACLTF	0.12	0.12	0.13	0.14	0.15	0.17	0.18	0.19	0.20	0.21	0.22	0.24	0.25
GDP g	14.7	14.5	14.2	13.9	13.5	13.1	12.7	12.3	11.8	11.4	10.9	10.3	9.79

Note: Numerical simulation of the solution of the model when the cost of external finance rises (δ from 0.20 to 0.44) and when the correlation between return on investment and short term foreign capital inflow rises (η from -0.01 to -0.13). Values of the short term foreign debt (STD) and the net (start up) investment (NI) are equal to average values reported in Table 1. Interest on short term foreign debt (ISTD), based on World Bank data set, is equal to average values of interest payment divided by the stock of short term foreign debt. Panels A and B report numerical results for the full sample in two different time periods (1990-99 and 2000-09). Panels report solutions for optimal ratios of reserves to short term foreign debt ratio (h*) and for the expected values of the following variables: variable investment (VI), total investment (TI), long term finance (LTF), average cost of long term finance (ACLTF), 2 years GDP growth (GDP g). Other parameters of the model are set as follows: short term foreign debt volatility, σ=0.8; scale parameter of long term finance cost function, s=0.0373; share of fixed capital (NI), α=0.25, and of other factors (VI), β=0.75; total factor productivity, ω=2.0628 in Panel A, ω=2.0842 in Panel B.

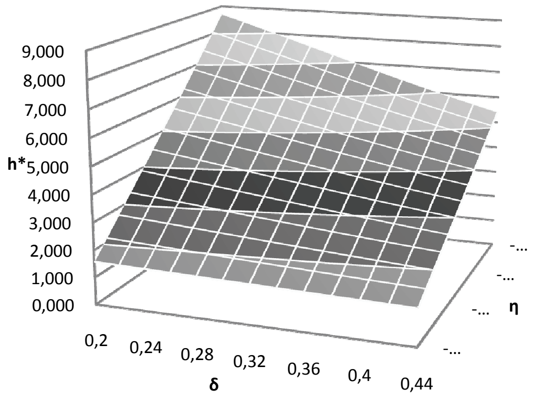
Source: Author's simulations.

According to the numerical simulations, thus, the model implies that the increasing level of reserves to short term foreign debt ratios is associated to the increasing competitiveness of domestic industry, which implies a negative relation (although weak) between capital inflow and output price. The higher (in absolute value) this relation is, the higher the extent of the unpredictable fluctuations of the most expensive sources of finance associated to net foreign capital inflow, thus the higher is the stock of reserves needed to stabilise expensive finance around its expected level. While affecting unexpected fluctuations of the examined variables, competitiveness of domestic industry does not affect expected levels, which are only determined by δ, as Table 3 clearly shows.

The determination of the value of h^* as a function of the two parameters δ and η is synthesised in Figure 12, taking data from the 2000-09 full sample's simulated solutions (panel B of Table 3). It can be observed that the optimal ratio h^* is a decreasing function of the marginal cost of long term finance, δ, and an increasing function of the competitiveness of the domestic industry (decreasing function of η), and that its sensitiveness on the level of δ is lower when η is closer to zero.

While Table 3 focuses on the functional link between the two parameters δ and η, Table 4 isolates the effect of different determinants of GDP growth. The values of the model's parameters are set to mimic precisely the observed changes in the GDP growth, the foreign reserves and the long term finance. This numerical exercise makes it is possible to extrapolate the values that, according to the model, the unobservable changes in the cost of long term finance and the total investment should take. Moreover, Table 4 makes it possible to separate and quantify the effects of three components on all changes occurring across the two decades. The first column associated to period 2000-09 simulates how the variables change as a simple effect of the observed change in the net investment (NI), the stock of short term foreign debt

(STD), and the slightly decreased level of the short term interest rate (r); the second column adds the effect of decreasing marginal cost of long term finance (δ); the third column adds the change in total factor productivity which is necessary, *ceteris paribus*, to reach the observed value of the GDP growth and long term finance.



Source: Author's simulation based on weighted sample average values during years 2000-09- Table 3, Panel B.

Figure 12 Optimal Reserves to Short Term Debt Ratio

The presented results emphasise that, according to the model, the lower cost of long term finance plays a very important role in the mechanism through which, in the last decade, BRIC economies have managed to almost double their average growth rates with little increase in new capital assets and short term foreign debt, and smaller economies have managed to increase growth despite a lower level of investment in new capital assets and a lower short term foreign debt. The value of δ compatible with the observed growth rates goes from 0.356 to 0.2415 in the weighted average sample (panel A), from 0.4505 to 0.3041 in the BRIC subsample (panel B), from 0.1915 to 0.122 in the smaller countries subsample (panel C). This change implies that the average cost of long term finance declines, in all countries, from nearly 14% to 10% while the quantity of long term finance increases from 39.5% to 61.25% of GDP. Comparing the two subsamples, the model's simulations underline that the declining average cost is a common trend in both subsamples but is sharper in the BRIC group (from 18.3% to 13.2%) than in the smaller countries group (from 7.8% to 6.1%). This is associated to almost a double quantity of long term finance in the BRIC economies (+87.13%) and a more modest change (+8.55%) in the smaller economies. Adding the net start up investment (given by data) and the variable investment (found as optimal solution) yields the total investment, which increases in all numerical simulations (+72% in BRIC and +5.8% in smaller economies, +46.6% on average).

Finally, to isolate the pure effect of the lower cost of long term finance, we compare the first and the second columns associated to the 2000-09 period. This cost effect determines 2.09 points of GDP growth in the full sample (whereas 1.6 points are due to the increased investment in new fixed assets and 1.21 points to changes in total factor productivity). In other words, the model generates almost half (42.7%) of the observed rise in GDP growth as a pure effect of the decreasing cost of long term finance. This pure cost effect determines more than half of GDP growth for the BRIC sample (63.11%) and one third for the smaller countries group (33.33%). The total factor productivity effect, in the third column, captures the residual growth that the model is not able to reproduce.

The simulations reported in Table 4 provide quantitative advice about this model's implications on the driving forces behind the emerging countries' observed growth rates. Clearly, an appropriate estimate of the impact of the cost of long term finance on economic growth should be carried out by jointly considering the effect of variables that in the theoretical model are assumed to be constant (see Section 2), which is beyond the scope of this work.

Table 4 Observed Variables as Optimal Solutions

Panel A Full Sample - Weighted Average

All countries	**1990-99**	**2000-09**		
STD to GDP	*5.67*	***4.57***		
NI to GDP	*13.60*	***16.39***		
η	-0.017	**-0.065**		
δ	0.356	0.356	**0.2415**	
s	0.0373	0.0373		
ω	2.0628	2.0628		**2.0842**
r	0.05	**0.04**		
RES to GDP	8.41	16.58	20.05	20.64
RES to STD (h*)	1.484	3.628	4.387	4.516
2y GDP growth	9.26	10.86	12.95	14.15
Δ growth		+1.60	+2.09	+1.21
LTF/GDP	39.5	45.85	59.02	61.25
AC of LTF	0.138	0.146	0.100	0.101
TI/GDP	52.82	62.06	75.22	77.46

Panel B BRIC Economies - Weighted Average

All countries	**1990-99**	**2000-09**		
STD to GDP	*3.09*	***4.07***		
NI to GDP	*14.11*	***19.22***		
η	-0.028	**-0.078**		
δ	0.4505	0.4505	**0.3041**	
s	0.0373	0.0373		
ω	2.1015	2.1015		**2.1086**
r	0.05	**0.04**		
RES to GDP	6.87	17.35	23.81	24.04
RES to STD (h*)	2.226	4.268	5.857	5.912
2y GDP growth	9.23	11.50	16.17	16.63
Δ growth		+2.27	+4.67	+0.46
LTF/GDP	34.12	42.57	63.12	63.84
AC of LTF	0.183	0.202	0.132	0.132
TI/GDP	48.07	61.63	82.18	82.90

Panel C Smaller Emerging Economies - Weighted Average

All countries	1990-99	2000-09		
STD to GDP	*8.81*	***5.26***		
NI to GDP	*12.99*	***12.16***		
η	-0.007	**-0.044**		
δ	0.1915	0.1915	**0.122**	
s	0.0373	0.0373		
ω	2.024	2.024		**2.0537**
r	0.05	**0.04**		
RES to GDP	10.23	13.32	14.10	14.62
RES to STD (h*)	1.161	2.533	2.682	2.779
2y GDP growth	9.10	8.54	8.89	10.15
Δ growth		-0.56	+0.35	+1.26
LTF/GDP	48.68	45.56	49.94	52.84
AC of LTF	0.078	0.078	0.060	0.061
TI/GDP	61.23	57.51	61.89	64.79

Note: Numerical simulation of the solution of the model given the values of the short term foreign debt (STD) and the net (start up) investment (NI). The parameters account for: correlation between return on investment and short term foreign capital inflow, η; marginal cost of long term finance, δ; scale parameter of long term finance cost function, s; total factor productivity, ω; interest rate on short term foreign debt, r (calculated from World Bank data set and equal to average values of interest payment divided by the stock of short term foreign debt). Panels report solutions for foreign reserves to GDP (RES to GDP), ratios of reserves to short term foreign debt ratio (RES to STD), and for the expected values of the following variables: 2 years GDP growth (2y GDP growth), change in growth (Δ growth), measured as the difference between the value of 2y GDP growth reported in one column and that reported in the column on the left; long term finance (LTF), average cost of long term finance (AC on LTF), total investment (TI). Other parameters of the model are set as follows: short term foreign debt volatility, σ=0.8; share of fixed capital (NI), α=0.25, and of other factors (VI), β=0.75.

Source: Author's simulations.

4. Conclusion

Traditional models explaining stockpiles of foreign reserves describe emerging countries as financing long term investment with volatile short term foreign capital and building reserves to cushion the real output of the economy in the event of foreign capital outflow. Foreign reserves, in that they channel domestic saving away from financing domestic investment, are often seen as causing high social costs or underinvestment in the emerging economies (Rodrik 2006; Greenwald and Stiglitz 2010). However, in the last decade, emerging economies kept short term foreign debt and investment in net fixed capital nearly unchanged (or even reduced them), but increased reserves disproportionately and simultaneously managed to increase GDP growth. This work has constructed a model that is able to derive both the high GDP growth and the growing stock of reserves as the result of the same process of the globalisation of the emerging economies.

The model derives the reserves to short term foreign debt ratio as an optimal choice of a Central Bank which acts in the interest of a representative firm. It builds on the assumption that short term finance, which depends on foreign capital inflow and domestic monetary policy, is volatile but cheaper, whereas long term private finance is more costly and the cost increases with quantity. A variable component of the investment is decided, and simultaneously long term finance is raised, once the available short term finance is known with certainty. The optimal solution depends crucially on two structural parameters newly introduced in this model and representative of the openness of the emerging countries to global markets: they account for the marginal cost of long term finance and for the competitiveness of the domestic industry.

An approximated analytical solution has been derived to carry out a sensitivity analysis of the model. The model implies that Central Banks of the emerging countries hold foreign reserves with the purpose of stabilising not the short term but the long term finance available to the firm around its expected level. For this purpose, a one-to-one Greenspan-Guidotti rule is not sufficient: emerging economies rationally accumulate higher ratios of reserves to short term foreign debt.

Numerical simulations of the model have reproduced the rising level of reserves in connection with rising GDP growth and rising long term finance as a result of two simultaneous changes in the aforementioned structural parameters: a falling cost of long term finance and increasing competitiveness of the domestic industry. Both changes could be ascribed to the higher openness of the emerging economies to global goods and financial markets. In particular, simulations have derived almost half of the observed rise in GDP growth during the decade 2000-09 as a pure effect of the decreasing cost of long term finance.

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Appendix: Proofs

Proposition 1

The sign of the ratio as a function of η is given by the sign of the factor multiplying the ratio $\frac{\eta}{D_0}$ of the RHS in equation (19). The expression $[(a-c)^2+3a^2\eta^2\sigma^2]$ on the numerator is always positive as it is a sum of squares. The expression $(a-c)$ in the denominator is always negative by the definitions of the parameters in (14) and (15). The expression $[1+d-\frac{bc}{a}+cD_0r]$ in the numerator is positive for values of parameters a, b, c and d consistent with the elasticity of the product to the variable investment calculated in $\bar{I}$, $e_I=\frac{a\bar{I}^2+b\bar{I}}{\frac{a}{2}\bar{I}^2+b\bar{I}+k}$. This can be seen by taking the expectations at time 0 of the optimal investment level from equation (17): the expected level of investment is $\bar{I}^e=\frac{(1+d)-b+cD_0r}{(a-c)}$; the expression $[1+d-\frac{bc}{a}+cD_0r]$ is hence positive if $\bar{I}^e(c-a)-b<-\frac{bc}{a}$, i.e. $\bar{I}^e<-\frac{b}{a}=\hat{I}$. This upper bound condition to the expected investment is not binding for values of the parameters consistent with a positive elasticity of the product to the investment: substituting $\bar{I}=\hat{I}$ into the expression for e_I, it turns out that $e_I=0$. Hence, the ratio that multiplies the parameter η is negative whenever the expression $((a-c)^2+3ac\eta^2\sigma^2)$ in the denominator is positive, i.e. whenever $\sigma^2<\sigma^{*2}=\frac{-(a-c)^2}{3\eta^2ac}$.

Proposition 2

Everything else being constant, a higher variance, σ^2, increases the value of the numerator of expression (19), as $3a^2\eta^2\sigma^2>0$, while decreasing the value of the denominator, as $3ac\eta^2\sigma^2<0$. Hence, for any value of $\eta<0$, the higher the variance, σ^2, the higher the value of h^*.

Proposition 3

From equation (19) the factors containing D_0 can be insulated: $\frac{[1+d-\frac{bc}{a}+cD_0r]}{D_0}=\frac{\left(1+d-\frac{bc}{a}\right)}{D_0}+cr$. This expression is positive (see proof of Proposition 1) and is lower as D_0 is higher. Hence, the value of h^* is closer to 1 as D_0 is higher, for any value of η.